

Violin Sounds and Experimental Setting for Teaching Advanced Calculus in a Developing Country

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Abstract—This paper describes a novel approach for teaching certain topics of Advanced Calculus to mechanical engineering students. The project was devised to cope with the lack of enthusiasm and engagement which usually resulted in very low performance at the end of the course. The idea was to introduce experimental analysis of sounds generated by a string musical instrument, so that the students had the opportunity to visualize what the solutions of partial differential equations predicted. A low-cost spectrum analyzer was built by programing an FPGA (Field Programmable Gate Array) board. Computer simulation was also included. Finally, a graduate student who is a concert violinist was invited to collaborate on the project. After two years of having started this approach, several positive results have appeared. Students have not only improved their performance, but have also managed to understand the importance of the concepts taught in this course. Furthermore, some students show empathy towards classical music, which they had never heard before.

Keywords—violin; sounds; calculus; spectrum; analysis

I. INTRODUCTION

Difficulties in learning certain topics of a subject might have different causes. Identifying and coping with those causes could be a hard task. In our institution, a compulsory two-semester Advanced Calculus (AC) course is offered to mechanical engineering students. One of the topics covered in this course is the solution of linear partial differential equations (PDEs), with special emphasis on physical applications. Two of such applications are the one-dimensional wave and heat conduction in a rod.

During the last years, and due to lack of basic prior knowledge, students have complained about not understanding the meaning of the coefficients that appear in the solutions of these PDEs, in the form of Fourier series.

It is well known that learning by doing helps students perform better in science. Students who physically experience scientific concepts understand them more deeply and score better on science tests, according to brain scans [1]. From our daily experience, we learn that visuals also play an important role in any learning process. In fact, the majority of students in this generation of technology and change are visual learners. They learn information best by seeing it in an example format, like a picture, map, graph, or video [2].

With all this in mind, the idea of introducing experimental analysis and computer simulations in the course syllabus finally took shape.

II. RESEARCH METHOD

A. Identifying Students' Weaknesses

At the beginning of the course, a simple question and answer class, along with a carefully designed placement test, revealed that almost all the students lacked prerequisite knowledge. Some physical and mathematical concepts of previous courses were inappropriately taught, or not taught at all. This situation kept the same for the last five years.

B. Difficulties in Understanding PDEs Solutions

The specific case of finding and understanding solutions of linear PDE's such as the one-dimensional wave equation, is briefly described below.

This physical phenomenon is described by the following PDE [3]:

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2} \quad (1)$$

The equation is subject to the boundary conditions:

$$y(0,t) = y(L,t) = 0 \quad (2)$$

The initial conditions are:

$$y(x,0) = f(x) \quad (3)$$

$$\frac{\partial y}{\partial t}(x,0) = g(x) \quad (4)$$

This problem can be solved via separation of variables [3]. Its solution expressed in Fourier series is given by (5).

$$y(x,t) = \sum_{n=1}^{\infty} A_n \cos \frac{n\pi at}{L} \sin \frac{n\pi x}{L} \quad (5)$$

The coefficients A_n are obtained as follows:

$$A_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx \quad (6)$$

In this way, we have arrived at the point that motivates this project, i.e., the physical interpretation of the coefficients A_n .

Before continuing with the description of the experimental framework, let us specify the physical meaning of the constants that appear in the vibrating string equation, i.e., a and L .

L is simply the length of the string and its unit is meters. The constant a is defined by (7).

$$a = \sqrt{\frac{T}{\mu}} \quad (7)$$

T is the tension in the string, and μ the string's linear density. According to SI units, T is measured in Newtons and μ is measured in kilograms per meter. Thus, a turns out to be the pulse speed, which is measured in meters per second [4].

Now, we are going to interpret these results in the context of musical stringed instruments, such as a violin or a guitar. The sound produced by the vibrating string consists of the superposition of infinitely many natural frequencies, each of which is determined by (8).

$$v_n = \frac{n}{2L} \sqrt{\frac{T}{\mu}} \quad (8)$$

The unit of measurement of frequencies is Hertz. It is easy to prove that the intensity of the sound produced depends on the sum of the square of each coefficient A_n [5]. More precisely, in the case of a sound wave, the intensity produced by the n th harmonic is given by (9).

$$I_n = 2\pi^2 u \rho v_n^2 A_n^2 \quad (9)$$

In this case, u is the speed of sound in meters per second and ρ is the air density in kilograms per cubic meter. The intensity is measured in Watts per square meter.

With the aim of getting the students to understand this dependence, the experimental framework was designed.

III. DESIGN AND IMPLEMENTATION OF A SPECTRUM ANALYZER

It is well known that there are some fundamental principles that must be followed in order to achieve the goal of effective teaching. Among them are the selection of appropriate bibliography and what the teacher imparts in the classroom. But in our case, it was clear that much more had to be done.

A. Selection of the FPGA Development Board

After a detailed research process, it was concluded that the hardware to acquire for our institution would be the Altera DE2-115 Development and Education Board [6]. This decision was made mainly on the basis that this teaching and project board is designed to meet educational needs, for both undergrad labs and undergrad/grad projects.

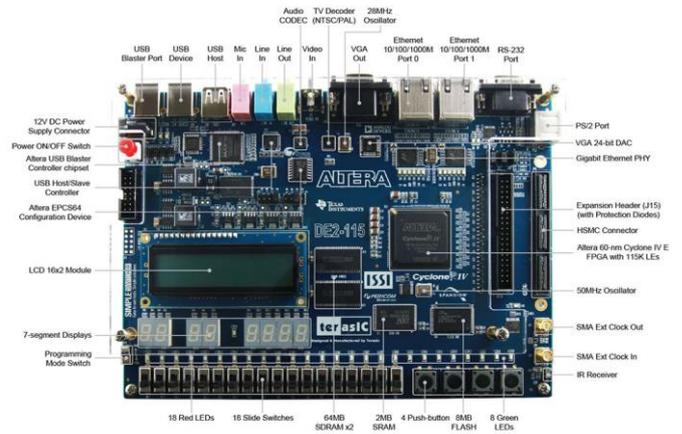


Fig. 1. Photograph of the DE2-115 Development Board

Fig. 1 shows a top view of the FPGA board chosen for this project. It depicts the layout of the board and indicates the location of the connectors and key components. Two important features to highlight in this board, is that it has a microphone-in jack and also a 24-bit encoder/decoder (CODEC), which makes the acquisition and programming of an analog-to-digital converter unnecessary. Below is a brief list of the hardware provided on the board.

- Altera Cyclone IV 4CE115 FPGA device
- 2MB SRAM
- Two 64MB SDRAM
- SD Card socket
- 50MHz oscillator for clock sources
- 24-bit CD-quality audio CODEC
- VGA DAC with VGA-out connector
- 114,480 Programmable Logic Elements (LEs)
- Switching and step-down regulators LM3150MH

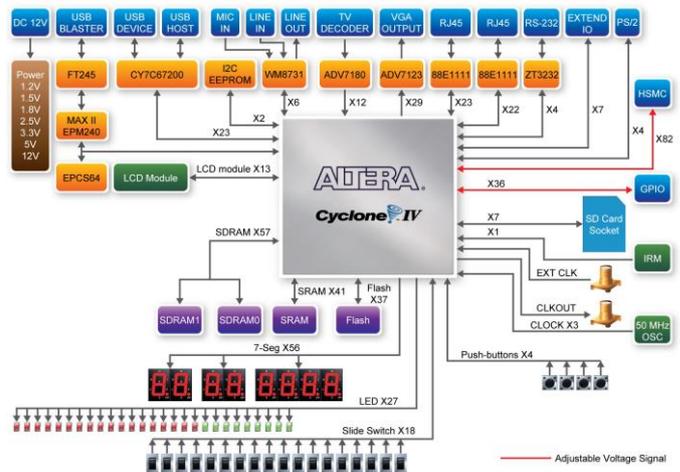


Fig. 2. Block Diagram of DE2-115 Development Board

Fig. 2 gives the block diagram of the DE2-115 board. It is important to note that all connections are made through the Cyclone IV FPGA device. Thus, it is easy for the user to configure the FPGA to implement any design.

B. Selection of the Microphone

This was a very particular phase in the process of implementing the experimental framework. In spite of the arduous research carried out to determine the best type of microphone to record violin sounds, we experienced a great disappointment knowing that none of these microphones could be imported into our country. In this way, we had to settle for using a low-cost condenser microphone, which adds some harmonic distortion to the sound emitted by the musical instrument. Our selection was the Takstar CM-60 condenser microphone.

C. Selection of the Display

At the beginning of the project, we thought about acquiring a display that was a fixed part of the experimental framework. At that time, our selection was the Terasic Multi-touch LCD Module Second Edition, which is an all-purpose capacitive touch-screen for FPGA applications. However, because our budget was very small, we had to acquire a common display for PC. Fig. 3 shows the block diagram of the experimental framework.

D. Programing the FPGA Development Borad

This stage of the project consists of obtaining, by means of some algorithm, the Discrete Fourier Transform (DFT) of the input signal digitized by the 24-bit audio CODEC. The standard algorithm is the Fast Fourier Transform (FFT). A good reference for understanding the principles of Digital Signal Processing (DSP) is [7].

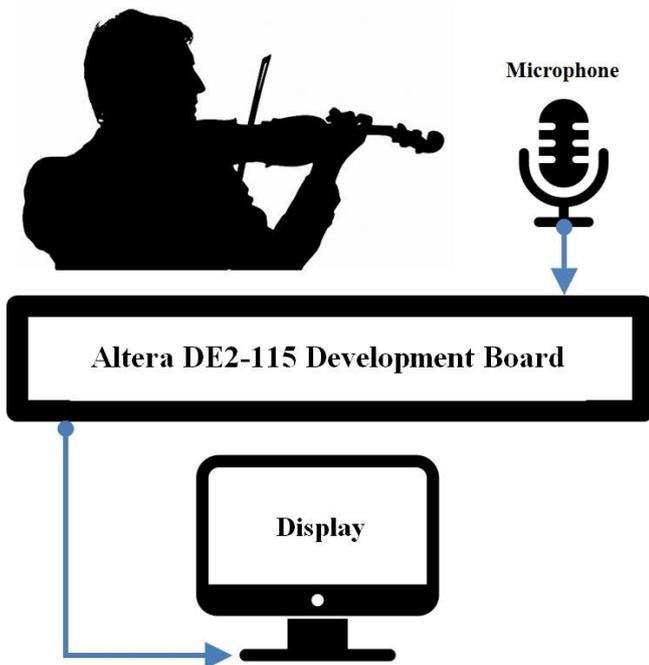


Fig. 3. Block Diagram of the Experimental Framework

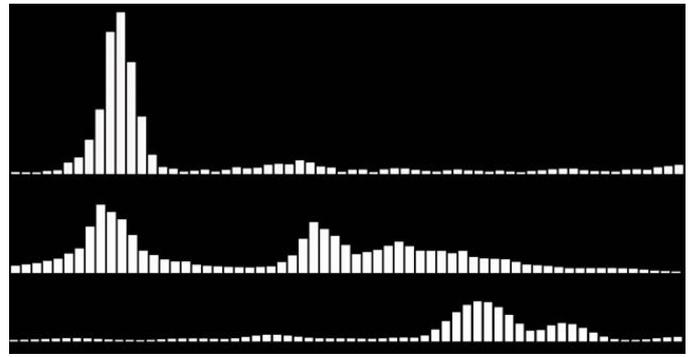


Fig. 4. Spectra of the tuning of a violin generated by the spectrum analyzer

The FFT algorithm allows us to move from the time domain to the frequency domain. The FFT of the violin sound is performed via the FFT MegaCore function, which is included in the software provided by Intel [8]. It is important to point out that the FFT IP core is a high performance, highly-parameterizable FFT processor. The FFT IP core implements a complex FFT or inverse FFT (IFFT) for high-performance applications. This core is built inside the Altera Cyclone IV FPGA device.

Fig. 4 shows some spectra corresponding to a typical violin tuning process. When the students saw these spectra generated by the analyzer, they were finally able to understand the physical meaning of the coefficients A_n that appear in the Fourier series given by (6). They could also understand that a spectrum is dynamic, that is to say, it varies with time.

IV. COMPUTER SIMULATIONS

In order to reinforce the learning achieved by the students, some computer simulations were carried out with MATLAB and also with free software.

Fig. 5 shows a violin tuning wave (.wav) file and its corresponding spectral analysis. In this case, the second graph is called a spectrogram. It contains information about time, frequency and amplitude in parametric form, that is to say, by using a color scale. Both graphs were obtained by using simple tools found on the website [9]. It is important to point out that this is a free website which contains a large number of audio files in different formats. Many other musical instruments can be picked from a huge data base, listened to and analyzed in a simple way.

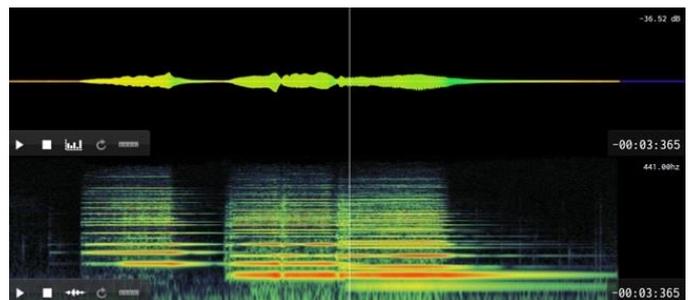


Fig. 5. Wave file and spectrogram of a violin tuning sound

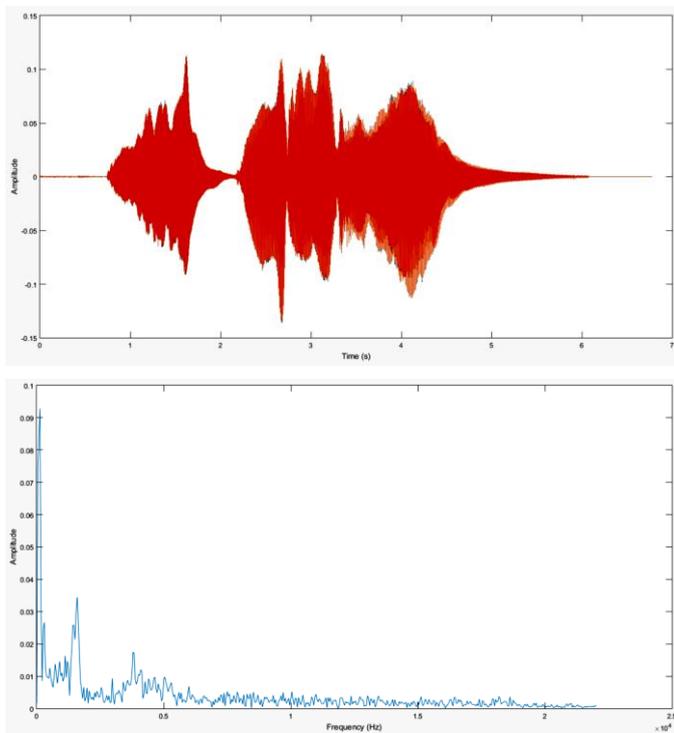


Fig. 6. Wave file plot and spectral analysis of a violin tuning sound

Fig. 6 shows the same results as Fig. 5, but this time using MATLAB. The script is the following:

```
>> [y,fs] = wavread('violin_tuning.wav');
>> t = linspace(0,length(y)/fs,length(y));
>> plot(t,y);
>> xlabel('Time (s)');
>> ylabel('Amplitude');
>> Nfft=1024;
>> f = linspace(0,fs,Nfft);
>> G = abs(fft(y,Nfft));
>> figure;
>> plot(f(1:Nfft/2),G(1:Nfft/2));
>> xlabel('Frequency (Hz)');
>> ylabel('Amplitude');
```

The wave file must be within the current path/folder that MATLAB is working in.

Another free, open-source, cross-platform audio software can be downloaded from [9].

Finally, for anyone who is searching for a solution to acoustic and vibration tests and measurements, then [10] is a good choice.

V. RESULTS

After applying these simple pedagogical strategies for one year, mechanical engineering students achieved a significant performance improvement. One could argue that this fact can be easily verified by comparing students' marks and average scores. But there are many other achievements that cannot be seen in numbers. Some of these achievements are: better attitude for learning, development of creative thinking and enthusiasm for intellectual growth.

The majority of the students who take this course come from low-income families and from small towns in the interior of our province. For this reason, by being part of this project, they find an opportunity to live in social equity and feel part of a culture of quality.

VI. CONCLUSIONS

In this paper, some pedagogical tools for teaching advanced calculus to mechanical engineering students have been presented and discussed. The design and implementation of a low-cost spectrum analyzer allowed students to understand concepts that, otherwise, might seem too mathematical or impractical. Special situations that appear as a result of belonging to a developing country make these simple tools very interesting.

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